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Quark Masses

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Because of the observed similarity between leptons and quarks, any lepton mass formula can be applied with proper modification to estimate the quark masses. In this work such an application of Rosen's (1978) lepton mass formula is attempted.

In leptoquark physics, the classifications of particles are made as follows: (i) e, ν_e, d, and u are the first-generation particles, (ii) μ, ν_μ, s, and c the members of the second generation, (iii) τ, ν_τ, b, and t those of the third generation, and (iv) δ, ν_δ, h, and g are the particles of the fourth generation. Bjorken (1978) has analyzed the masses of these particles as function of the generation number and charge, and adopted the following masses for the quarks:

$$m_u \sim 4 \text{ Mev}, \qquad m_d \sim 7 \text{ Mev}, \qquad m_s \sim 150 \text{ Mev},$$
$$m_c \sim 1.2 \text{ Gev}, \qquad m_b \sim 4.6 \text{ Gev} \tag{1}$$

On the other hand, Pakvasa and Sugawara (1979) have obtained the following relations between the masses of the quarks and leptons by using the permutation symmetry for Higgs couplings:

$$m_\mu / m_\tau = m_s / m_b = m_c / m_t$$
$$m_\tau / m_\delta = m_b / m_h = m_t / m_g \tag{2}$$

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Barut (1979) has suggested a lepton mass formula. If n is the quantum number corresponding to the $(n+1)$th generation lepton such that $n = 0, 1, 2, 3 \ldots$, respectively, for $e, \mu, \tau, \delta, \ldots$ lepton etc., the mass formula can be written as

$$m_{n+1} = m_1\left[1 + \lambda\left(1^4 + 2^4 + \cdots + n^4\right)\right], \qquad \lambda = \tfrac{3}{2}\alpha^{-1} \tag{3}$$

where, m_{n+1} is the mass of $(n+1)$th generation lepton, α is the fine-structure constant, $m_1 = m_e$ is the mass of the first-generation $(n = 0)$ lepton.

This formula predicts the value of $m_\tau = 1786$ Mev which agrees extremely well with the experimental value of 1787 Mev. Also the predicted value of $m_\delta = 10294$ Mev agrees well with that predicted by Bjorken (1978). Thus the success of Barut's formula is remarkable. Minami and Nakashima (1979) have applied Barut's formula to quark masses with suitable choice of the value of parameter λ [e.g., $\lambda = \frac{9}{4}\alpha^{-1}$ or $\frac{14}{5}\alpha^{-1}$, $m_1 = m_u$ for $\frac{2}{3}$ charged quarks and $\lambda = \frac{1}{4}\alpha^{-1}$, $m_1 = m_d$ for $1/3$ charged quarks] and the predicted values of the masses of the third and fourth generation quarks agree well with those given by Bjorken and other sources. The formulas for leptons and quarks also exactly satisfy the mass ratio relation (2). The predicted value of the first and second ratio in equation (2) are 0.059 and 0.17, respectively. The experimental value of m_μ/m_τ agrees with the former value well.

Rosen (1978) has derived a different lepton mass formula from the idea of equipartition which he had proposed earlier (1971). His theory includes QED self-energy and gravitational self-energy. The theory gives electron mass in terms of fine-structure constant and gravitational constant, and the theoretical value agrees very well with the experimental one. He extends his theory to include μ and heavier leptons by introducing a principal quantum number n with $n = 0, 1, 2, 3, \ldots$, respectively, for $e, \mu, \tau, \delta, \ldots$ lepton etc. Thus if n is the quantum number corresponding to the $(n+1)$th generation lepton, Rosen's formula can be written as

$$m_{n+1} = m_1\left[1 - \tfrac{1}{4}f(n)\right]\exp\left[\tfrac{1}{9}\pi\alpha^{-1}f(n)\right] \tag{4}$$

where $f(n) = n(n^{1/2} + 2)^{-2}$ and α is the fine-structure constant.

From equation (4)

$$\frac{m_{n+1}}{m_1} = \begin{array}{cccc} n=0 & n=1 & n=2 & n=3 \\ 1 & 206.2 & 3748 & 3.066\times 10^4 \end{array} \tag{5}$$

The predicted value of m_μ agrees well with the experimental value. However, the predicted value of $m_\tau = 1912$ Mev differs considerably from the experimental value of 1787 Mev. The predicted value of $m_\delta = 15.6$ Gev is about one and a half times larger than that given by Barut's formula. Even with considerable disagreement for the third and fourth generation leptons, it should be noted that Rosen's formula has a theoretical basis.

If we apply Rosen's formula (4) as such for 2/3 and 1/3 charged quarks separately with m_1 replaced by m_u and m_d, respectively, we find that the mass ratio relation (2) is exactly satisfied. From (5) then

$$m_\mu/m_\tau = m_s/m_b = m_c/m_t = 0.055$$

$$m_\tau/m_\delta = m_b/m_h = m_t/m_g = 0.122 \tag{6}$$

From (6) we see that compared with the experimental value of m_μ/m_τ, the theoretical value is somewhat low. Also the second ratio is less than that given by Barut's formula. Moreover, applying Rosen's formula to 2/3 and 1/3 charged quarks separately with $m_u = 4$ Mev and $m_d = 7$ Mev as given in (1), we see considerable disagreement with the values predicted by Minami and Nakashima.

However, an improvement of agreement for quark masses can be obtained by modifying (4) as, for $n \geqslant 1$

$$m_{n+1} = Am_1[1 - \tfrac{1}{4}f(n)]\exp[\tfrac{1}{9}\pi\alpha^{-1}f(n)] \tag{7}$$

where A is a constant. The value of A can be obtained by fitting m_1 and m_2. Thus with the values $m_u = 4$ Mev and $m_c = 1.2$ Gev as given in (1), we have $A = 1.46$ for 2/3 charged quarks. Again with $m_d = 7$ Mev as in (1) and $m_s = 246$ Mev as taken by Minami and Nakashima (1979) [because Bjorken's value $m_s = 150$ Mev does not satisfy (2) well], we have $A = 0.17$ for 1/3 charged quarks. In the Table I we give the predicted values in Gev of the masses of the third and fourth generation quarks according to (4) and (7) and compare them with those predicted in Minami and Nakashima (1979).

In conclusion, it should be noted that whereas $A = 1$ for leptons (unit charge), $A = 1.46$ for 2/3 charged quarks and $A = 0.17$ for 1/3 charged

Table I. Mass of Quarks in Gev

Source	m_t	m_g	m_b	m_h
Equation (4)	16	123	26	213
Equation (7)	22	179	4.5	37
Minami and Nakashima (1979)	21	121	4.1	24

quarks. The situation is similar in the case of the modification made by Minami and Nakashima (1979) in Barut's formula ($\lambda = \frac{3}{2}\alpha^{-1}$, $\lambda = \frac{9}{4}\alpha^{-1}$ or $\frac{14}{5}\alpha^{-1}$ and $\lambda = \frac{1}{4}\alpha^{-1}$, respectively, for leptons, 2/3 charged quarks and 1/3 charged quarks). Although, the values of A (or λ) do not reflect any systematic dependence on the magnitude of the charge of the particle, it seems that any lepton mass formula with suitable modification can provide tentative masses of third and fourth generation quarks.

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